Practice problems

IE 5545: Decision Analysis
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1. (Tadelis 8.10) Consider the following dynamic game: Player 1 can choose to play it safe (S), in which case both he and player 2 get a payoff of 3 each, or he can risk (R) playing a game with player 2. If he chooses R, the two players play the following simultaneous-move game:

	A	B
C	(8,0)	(0,2)
D	(6,6)	(2,2)

   (a) Draw a game tree that represents this game. How many proper subgames does it have?
   (b) Are there any other game trees that also represent this game? Explain why or why not.
   (c) Provide the strategic form of the dynamic game.
   (d) Find all Nash and subgame-perfect Nash equilibria of the dynamic game.

2. Consider a zero sum game with the following payoff matrix for player I:

$$A = \begin{bmatrix} -1 & 7 & -4 & 2 \\ 3 & -3 & 5 & -1 \end{bmatrix}.$$

   (a) Restricting attention to pure strategies, compute the maximin strategy and the maximin value for both players I and II. For the rest of the problem, we focus on mixed strategies. Denote player I's mixed strategy as $\sigma_1 = [p \quad 1-p]^\top$ for some $p \in [0,1]$.
   (b) We are interested in computing player I's maximin strategy, and the corresponding maximin value. Write down an optimization problem in p whose optimal value is the maximin value of player I, and whose solution p^* corresponds to her maximin strategy.
   (c) Solve the preceding problem (graphically or otherwise), and find the value of p^* and the maximin value for player I.
   (d) Show that the maximin strategy for player II is of the form $\sigma_2 = [0 \;\; 0 \;\; q \;\; 1-q]^\top$ for some $q \in [0,1]$.

3. Recall the infinitely repeated Prisoner's Dilemma $G(\infty, \delta)$, with discount factor $\delta < 1$, where G is given by the following strategic form:

	S	B
S	$-1,-1$	$-10,0$
B	$0,-10$	$-5,-5$

The two players play G repeatedly at times $t = 0, 1, 2, \ldots$. The payoff of player i is given by

$$\pi_i(s_i, s_{-i}) = \sum_{t=0}^{\infty} \delta^t \pi_i^G(a_i^t, a_{-i}^t).$$

Consider the following modification of the grim-trigger strategy for player 1. Player 1 starts by playing S (stay silent) and continues doing so until player 2 first plays B (betray). Thereafter, player 1 continues playing B, until the first time player 2 plays S, at which point player 1 forgives player 2 and starts playing S (until the next betrayal by player 2 when the same cycle of punishment and forgiveness continues). We will call such a player 1 a *forgiver*. We are interested in the case where both players are forgivers and ask when such strategies form an SPNE.

In order to define the strategy formally, we need to define a (common) state of the game X_t at time t to denote whether the game is in cooperation mode ($X_t = 0$), or whether player 1 is being punished ($X_t = 1$) or whether player 2 is being punished ($X_t = 2$). The state evolves as follows: First, the game starts in cooperation mode: $X_0 = 0$. For any $t \geq 1$, we have

$$X_{t+1} = \begin{cases} 0, & \text{if } X_t = 0, \text{ and } (S,S) \text{ or } (B,B) \text{ was played at time } t; \\ 1, & \text{if } X_t = 0, \text{ and } (B,S) \text{ was played at time } t; \\ 2, & \text{if } X_t = 0, \text{ and } (S,B) \text{ was played at time } t; \\ \\ 0, & \text{if } X_t = 1, \text{ and } (S,B) \text{ was played at time } t; \\ 1, & \text{if } X_t = 1, \text{ and } (S,B) \text{ was } \textbf{not played} \text{ at time } t; \\ \\ 0, & \text{if } X_t = 2, \text{ and } (B,S) \text{ was played at time } t; \\ 2, & \text{if } X_t = 2, \text{ and } (B,S) \text{ was } \textbf{not played} \text{ at time } t. \end{cases}$$

(In words, if the game is in cooperation mode, and (S,S) or (B,B) is played, then the game stays in cooperation mode. Otherwise, if (B,S) is played, then game transitions to punishing player 1 or if (S,B) is played, the game transitions to punishing player 2. If player 1 is being punished, that remains the case until (S,B) is played, whereupon the game transitions to cooperation mode. Similarly, if player 2 is being punished, that remains the case until (B,S) is played, whereupon the game transitions to cooperation mode.)

Define $\mathsf{forgiver}_1$ to be the following strategy for player 1: play S if $X_t = 0$ or $X_t = 1$, and play B if $X_t = 2$.

Similarly, define $\mathsf{forgiver}_2$ to be the following strategy for player 2: play S if $X_t = 0$ or $X_t = 2$, and play B if $X_t = 1$.

(a) State the path of play (the sequence of action profiles at each time) under the strategy profile $(\mathsf{forgiver}_1, \mathsf{forgiver}_2)$. What is player 1's expected payoff under this strategy profile? (Your answer will be in terms of δ.)

(b) Suppose the state at time t is given by $X_t = 0$. What is the path of play under the one-step deviation for player 1? What is player 1's expected payoff under this deviation in this subgame?

(c) Suppose the state at time t is given by $X_t = 1$. What is the path of play under the strategy profile $(\mathsf{forgiver}_1, \mathsf{forgiver}_2)$ and under the one-step deviation for player 1? What is player 1's expected payoff for each case in this subgame?

(d) Suppose the state at time t is given by $X_t = 2$. What is the path of play under the strategy profile $(\mathsf{forgiver}_1, \mathsf{forgiver}_2)$ and under the one-step deviation for player 1? What is player 1's expected payoff for each case in this subgame?

(e) For what values of $\delta \in (0, 1)$ is the strategy profile $(\mathsf{forgiver}_1, \mathsf{forgiver}_2)$ an SPNE of $G(\infty, \delta)$? (Be sure to state the *principle* you are using to conclude the result.)

4. Consider a two player static game of incomplete information, where each player $i \in \{1, 2\}$ has a type $\theta_i \in [0, 1]$ that is drawn independently and identically from a uniform distribution over $[0, 1]$. Each player i, after learning her type, chooses an action $x_i \geq 0$. The payoff function of player i is given by

$$\pi_i(x_i, x_j, \theta_i, \theta_j) = \theta_i x_i x_j - x_i^3,$$

where $j \neq i$. Your goal is to find a Bayes-Nash equilibrium for this game.

(a) Give a formal description of a pure strategy for player i.

(b) Suppose player $j \neq i$ is playing a fixed pure strategy s_j. What is the interim expected payoff $\Phi_i(x_i|\theta_i)$ of player i when her type is θ_i and her action is x_i?

(c) State what conditions Φ_1 and Φ_2 must satisfy in a Bayes-Nash equilibrium?

(d) Suppose player $j \neq i$ is playing a fixed pure strategy s_j. What is the best response strategy for bidder i?

(e) Find a Bayes-Nash equilibrium for this game.